ON THE USE OF GENERAL NON-LINEAR DETECTOR FOR A DELAY LOCKED LOOP

S.Thayaparan, T.S.Ng and J.Wang

Department of Electrical & Electronic Engineering
The University of Hong Kong
Hong kong
Email: sthay@eee.hku.hk

ABSTRACT

This paper presents the performance comparison on the use of general non-linear detectors in a non-coherent delay locked tracking scheme for direct-sequence spread-spectrum systems. In the delay locked loop (DLL), the square law-envelope detectors are replaced by a full wave (even) n^{th}-law detectors and this scheme is analysed in the presence of additive white gaussian noise. The tracking error of the DLL is compared for different values of n (n=0.5, 1, 2, 3 respectively). The results shows that the tracking error of the DLL can be reduced further in low signal to noise power ratio region by using the full wave (even) n^{th}-law detectors for $n<2$ when compare with using the square law envelope detectors ($n=2$).

1. INTRODUCTION

The direct sequence spread spectrum (DS/SS) systems have become the popular choice for commercial applications such as mobile communications and positioning systems. In such a DS/SS communication system, synchronising the locally generated PN code sequence to the PN code sequence in the received signal is a crucial aspect, specially in low signal to noise power ratio. The synchronisation process is generally to first acquire the rough timing (code acquisition) and then track the acquired code with fine timing (code tracking). The delay lock technique is the widely accepted method for tracking purposes. The tracking error and the mean time to lose lock are two important criteria used to measure the DLL performance. Small tracking error increases the mean time to lose lock. During normal operations, low bit error rate transmission is guaranteed when the tracking error is small in the DLL. The occasional signal fading may result in sudden reduction in the signal to noise power ratio and this may cause the out of lock situation which requires the reacquisition process. In the delay locked loop, the incoming PN code is correlated with an early (advanced one-half chip) and a late (retarded one-half chip) version of the locally generated PN code. The error signal is obtained by the difference of the two demodulated correlation signals. In the traditional non-coherent DLL, the square law-envelope detectors (SLED) are used to generate the error signal. The square law-envelope detector is a special case of the full-wave (even) n^{th}-law detector when $n=2$. In this paper, the performance of the DLL is analysed using the full-wave (even) n^{th}-law detectors. The tracking error of the DLL is compared for four different values of n ($n=0.5, 1, 2, 3$ respectively). The case $n=1$ is specially interesting because of its linear loop characteristic curve.

2. ANALYSIS

In a direct sequence spread spectrum (DS/SS) system, the received signal which is corrupted by the additive white noise can be written as

$$r(t) \;=\; \sqrt{2P}C(t)d(t)\cos(\omega_c t+\phi) \;+\; n(t) \qquad (1)$$

where P is the received signal power, $d(t)$ is the data sequence, $C(t)$ is the spreading code, ω_c and ϕ are the angular carrier frequency and phase, respectively, and $n(t)$ is the additive white noise.

In the delay locked loop, the spacing between the early and late replicas of the locally generated PN codes is considered to be one chip time T_c and this spacing is preferred from the point of view of acquisition. For simplicity, no data modulation is considered in this analysis. The following results are therefore applicable to the ranging applications or the DS/SS systems that employs a pilot channel for synchronisation. Fig. 1 shows the block diagram of the DLL. The normalised phase error between the received PN code sequence and the locally generated PN code sequence is denoted by ε.

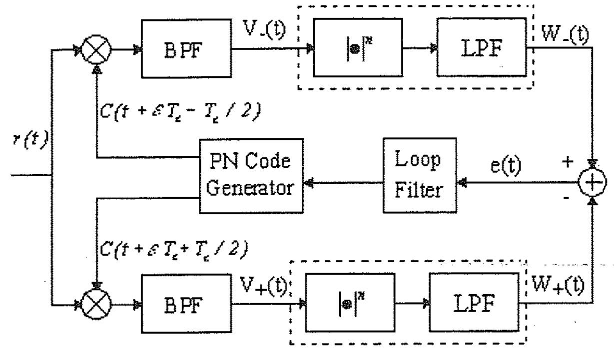

Fig. 1: Delay lock Loop with full-wave (even) n^{th}-law detectors

Since the code rate $(1/T_c)$ is much larger than the loop bandwidth, the code self noise is neglected. The operation of the tracking loop is the same as that of the conventional non-coherent DLL. Referring to Fig.1,

$$V_\pm(t) = \sqrt{2P}\;R(\varepsilon \pm \frac{1}{2})\cos(\omega_c t+\phi) + n_w(t) \qquad (2)$$

The noise component $n_w(t)$ is the band pass process of additive white noise given by,

$$n_w(t) = \sqrt{2}\{N_{c\pm}(t)\cos(\omega_c t+\phi) - N_{s\pm}(t)\sin(\omega_c t+\phi)\}$$

where $N_{c\pm} = n_c(t)C(t+\varepsilon T_c \pm \frac{T_c}{2})\otimes h_l(t)$ and

$$N_{s\pm} = n_s(t)C(t+\varepsilon T_c \pm \frac{T_c}{2})\otimes h_l(t)$$

The equivalent low pass filter impulse response of the band pass filter and the convolution are denoted by $h_l(t)$ and $\otimes$ respectively.

The noise components $n_c(t)$ and $n_s(t)$ are statistically independent, stationary white gaussian noise processes of density $N_o/2$ (double sided) and one sided bandwidth $B/2$, where the arm filter bandwidth is B. The auto-correlation function $R(\varepsilon)$ of the spreading PN sequence is approximated by

$$\begin{aligned} R(\varepsilon) &= E\{C(t)C(t+\varepsilon T_c)\} \\ &= \begin{cases} 1-|\varepsilon|, & |\varepsilon|\le 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \qquad (3)$$

For long PN sequences, this approximation can be relaxed. The non-linear detector output $W_\pm(t)$ can be realised as the low pass process of $|V_\pm(t)|^n$. The normalised characteristic curve of the model is obtained in the absence of noise.

$$W_\pm(t) = C(n)\{\sqrt{2P}R(\varepsilon \pm \frac{1}{2})\}^n \qquad (4)$$

where $C(n) = \dfrac{\Gamma(n+1)}{2^n\Gamma^2(1+\frac{n}{2})}$

It follows that the error signal

$$e(t) \;=\; \left(\sqrt{2P}\right)^n C(n)\left\{R^n(\varepsilon-\frac{1}{2}) - R^n(\varepsilon+\frac{1}{2})\right\} \qquad (5)$$

The normalised characteristic curves for the values of n, 0.5, 1, 2 and 3 are plotted in Fig. 2.

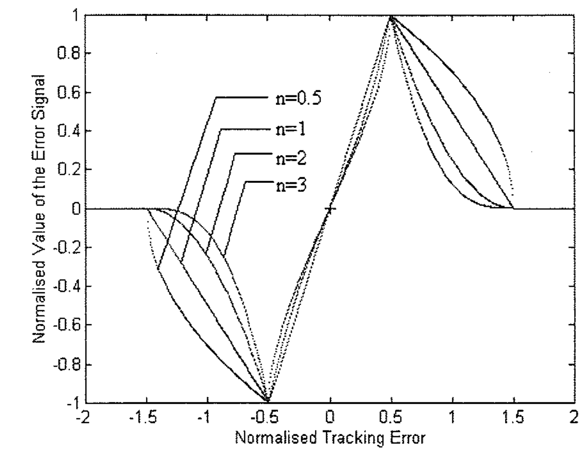

Fig. 2: The normalised characteristic curves

In practice, linearity of the normalised characteristic curve in the region $|\varepsilon| \le \frac{1}{2}$ is preferred. From Fig. 2, the values of n that satisfy this condition are $n=1$ and $n=2$. The auto-correlation function of the noise at the output of the non-linear detectors can be obtained following the analysis in [3], chapter 13.

$$R_w(\tau) = \sum_{m=0}^{\infty}\sum_{\substack{k=0 \\ \text{for even } m+k}}^{\infty} \frac{\varepsilon_m h^2_{mk}}{k!} R^k{}_n(\tau)\cos m\omega_c\tau \qquad (6)$$

where the Neumann factor $\varepsilon_m = \begin{cases} 1, & m=0 \\ 2, & m=1,2,\ldots \end{cases}$

and $$h_{mk} = \frac{\Gamma(n+1)(P/4\sigma^2)^{\frac{m}{2}}}{m!\,\Gamma[1-(m+k-n)/2](\sigma^2/2)^{\frac{k-n}{2}}}\;{}_1F_1(\frac{m+k-n}{2};m+1;-\frac{P}{4\sigma^2})$$

The factor ${}_1F_1(a;c;z)$ is the confluent hypergeometric function, $\sigma^2 = N_0B$ is the variance of the band pass additive white noise and $R_n(\tau) = E\{n_w(t)n_w(t+\tau)\}$. Since we use the full-wave (even) detector, the coefficient h_{mk} vanishes when $m+k$ is odd. The auto-correlation function $R_n(\tau)$ of the band pass process of gaussian noise $n_w(t)$ can be expressed as,

$$R_n(\tau) = 2R_{nc}(\tau)\cos w\tau \tag{7}$$

where $R_{nc}(\tau) = E\{N_c(t)N_c(t+\tau)\}$

By combining the above expressions (6) and (7), and since the non linear detector output is a low-pass process, after some mathematical manipulations the auto correlation function $R_e(\tau)$ of the error signal $e(t)$, when the phase error $\varepsilon \approx 0$, becomes,

$$R_e(\tau) = \sum_{k=1}^{\infty}\frac{1}{k!}\left\{4h_{kk}^2 + 2^k I_k h_{ok}^2\right\} R^k{}_{nc}(\tau) \tag{8}$$

where $I_k = \dfrac{k-1}{k} I_{k-2}$ and $I_0 = 2,\ I_1 = 0$.

The power spectral density $S_e(t)$ of $e(t)$ when $\varepsilon \approx 0$ is the Fourier transform of $R_e(\tau)$.

$$S_e(f) = \sum_{k=1}^{\infty}\frac{1}{k!}\left\{4h_{kk}^2 + 2^k I_k h_{ok}^2\right\} {}_kS_{nc}(f) \tag{9}$$

where ${}_kS_{nc}(f) = {}_{k-1}S_{nc}(f) \otimes S_{nc}(f)$,

${}_1S_{nc}(f) = S_{nc}(f)$ and $S_{nc}(f)$ is the power spectral density of $n_c(t)$.

The tracking error σ_m of the delay locked loop is obtained analytically by means of the standard linearised loop analysis.

$$\sigma_m = \sqrt{\frac{2S_e(0)B_L}{K^2{}_L}} \tag{10}$$

where $K_L = \dfrac{4n\ \ \Gamma(n+1)(\sqrt{2P})^n}{2^{2n}\Gamma^2(1+\frac{n}{2})}$.

The loop noise bandwidth and the loop gain is denoted as B_L and K_L respectively. Let us denote the ratio of the arm filter bandwidth to the loop noise bandwidth be $\gamma = \dfrac{B}{B_L}$. The tracking error of the DLL depends on the value of γ. Fig. 3 demonstrates the tracking error of the DLL for the values of n = 0.5, 1, 2 and 3, respectively and these curves are plotted for the expression in (10) and the value of γ = 20 dB. These performance curves show a considerable difference in the low SNR region even though all four cases perform almost equally in high SNR. When $n < 2$, the ratio of the effective error signal power to noise power is reduced considerably in low SNR and this enable us to have smaller tracking error.

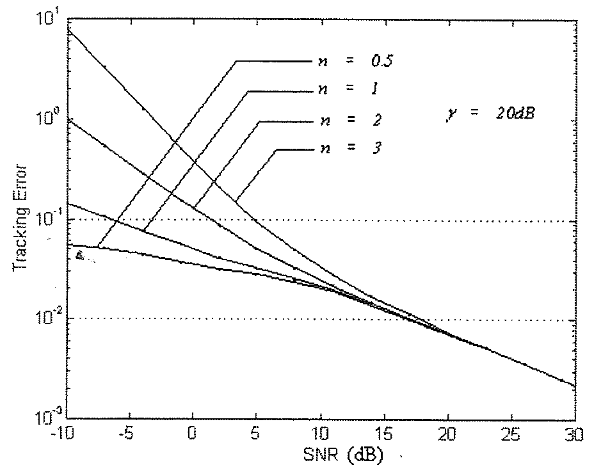

Fig. 3: The performance curves of the DLL

The curves plotted in Fig. 3 may vary from the actual results in very low SNR because we use the standard linearised loop analysis to determine the tracking error. However, in such a situation, the actual tracking error will generally be lower than that obtained by linear loop analysis. The results obtained by linear analysis is generally considered as a good approximation of the actual performance in the region of SNR which we are interested in.

3. CONCLUSION

The paper has presented an analysis of the performance of using general non-linear detectors in a non-coherent DLL tracking scheme for DS/SS systems. The tracking error of the DLL is compared for the cases of $n = 0.5,\ 1,\ 2$ and 3. The results showed that tracking error of the non-coherent delay locked loop can be reduced by using the full-wave (even) n^{th}-law detectors for $n < 2$ when compare with that using the square law envelope detectors in low signal to noise power ratio.

ACKNOWLEDGMENT

This work was supported by the Research Grants Council (RGC) of The Hong Kong Government and the CRCG of The University of Hong Kong.

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